LECTURE 2

[i]

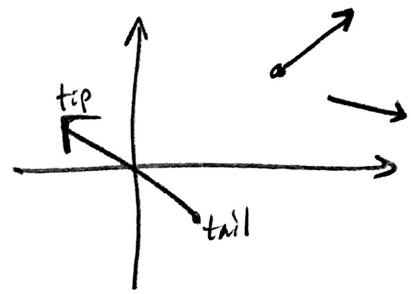
LAST TIME : $\mathbb{R}^n = \{(x_1,\dots,x_n) \mid x_1,\dots,x_n \in \mathbb{R}\}$

$\mathbb{R}^2$ = plane , $\mathbb{R}^3$ = 3-space , DISTANCE : $P=(x_1,\dots,x_n)$, $Q=(y_1,\dots,y_n)$

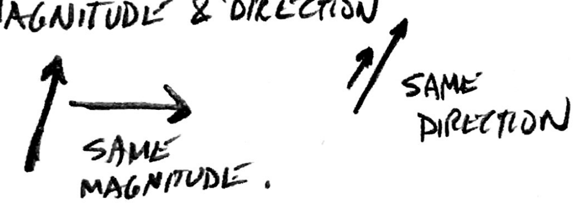
$$|PQ| = \sqrt{(x_1-y_1)^2 + \dots + (x_n-y_n)^2}$$

VECTORS IN $\mathbb{R}^2$:

ARROWS



– REPRESENT QUANTITIES w/ MAGNITUDE & DIRECTION

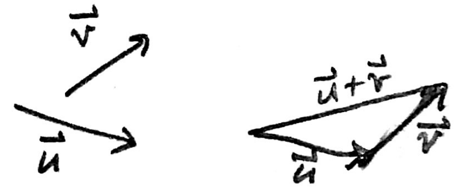


DECLARE TO VECTORS TO BE EQUAL IF THEY HAVE THE SAME MAGNITUDE & DIRECTION.

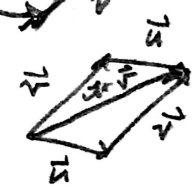
NOTATION : • $\vec{v}$
• (bold) v – book.

ARITHMETIC :

ADDITION :



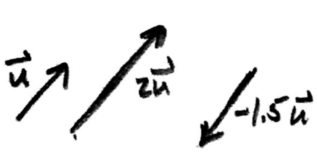
(PARALLELOGRAM RULE :



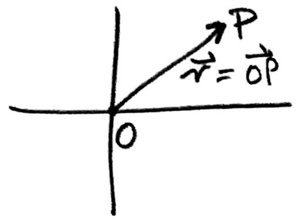
)

SCALAR MULTIPLICATION : $c \in \mathbb{R}$, $\vec{v}$ a vector

$c\vec{v}$ = SAME DIR. AS $\vec{v}$, MAG. SCALE BY c
(OPP. DIR. IF $c<0$)



EVERY VECTOR IS EQUAL TO ONE W/ TAIL AT O:



VECTORS IN $\mathbb{R}^2 \longleftrightarrow \mathbb{R}^2$

$\overrightarrow{OP} \longleftrightarrow P$

$\parallel$

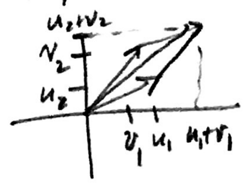
<u>POSITION VECTOR</u> OF P

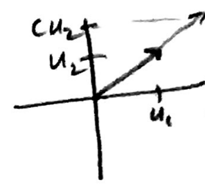
(WE CONSIDER VECTORS & POINTS <u>DIFFERENT</u>)

IF $P = (v_1, v_2)$, WE WRITE $\overrightarrow{OP} = \langle v_1, v_2 \rangle$

<u>ARITHMETIC</u> (AGAIN): $\vec{u} = \langle u_1, u_2 \rangle$, $\vec{v} = \langle v_1, v_2 \rangle$, $c \in \mathbb{R}$:

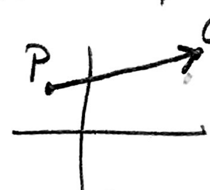
- $\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$
- $c\vec{u} = \langle cu_1, cu_2 \rangle$



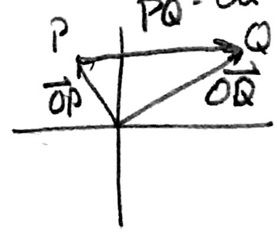


<u>DISPLACEMENT</u>: $P, Q \in \mathbb{R}^2$, THE <u>DISPLACEMENT VECTOR</u> FROM P TO Q

IS $\overrightarrow{PQ}$ = VECTOR W/ TAIL AT P, TIP AT Q



NOTE: $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$



$\therefore \quad \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

<u>MAGNITUDE</u>: $\vec{v} = \langle v_1, v_2 \rangle$, THEN

MAGNITUDE OF $\vec{v} = |\vec{v}| = \sqrt{v_1^2 + v_2^2}$ (ALSO WRITE $\|\vec{v}\|$)

HAVE A ZERO VECTOR $\vec{0} = \langle 0, 0 \rangle$; SOME BASIC PROPERTIES:

(FOLLOW FROM ADD. & MULT. PROPS IN $\mathbb{R}$)

- $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
- $\vec{u} + \vec{0} = \vec{u}$
- $\vec{u} + (-\vec{u}) = \vec{0}$
- $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$
- $(c+d)\vec{u} = c\vec{u} + d\vec{u}$
- $(cd)\vec{u} = c(d\vec{u})$
- $1\vec{u} = \vec{u}$

VECTORS IN $\mathbb{R}^3$: ARROWS ...

VECTORS IN $\mathbb{R}^3 \longleftrightarrow \mathbb{R}^3$

$\overrightarrow{OP} \longleftrightarrow P$

$\vec{u} = \langle u_1, u_2, u_3 \rangle$, $\vec{v} = \langle v_1, v_2, v_3 \rangle$, $c \in \mathbb{R}$

$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$, $c\vec{u} = \langle cu_1, cu_2, cu_3 \rangle$

... SAME IN $\mathbb{R}^n$ ...

STANDARD BASIS VECTORS :

$\mathbb{R}^2$: $\vec{i} = \langle 1, 0 \rangle$, $\vec{j} = \langle 0, 1 \rangle$ ... $\vec{u} = \langle u_1, u_2 \rangle = u_1 \vec{i} + u_2 \vec{j}$.

$\mathbb{R}^3$: $\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$, $\vec{k} = \langle 0, 0, 1 \rangle$ ... $\vec{u} = \langle u_1, u_2, u_3 \rangle = u_1 \vec{i} + u_2 \vec{j} + u_3 \vec{k}$.

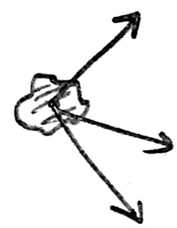
$\mathbb{R}^n$: $\vec{e}_1 = \langle 1, 0, 0, \ldots, 0 \rangle$, $\vec{e}_2 = \langle 0, 1, 0, \ldots, 0 \rangle$, ..., $\vec{e}_n = \langle 0, 0, \ldots, 0, 1 \rangle$

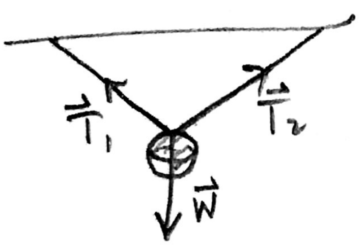
$\vec{v} = \langle v_1, \ldots, v_n \rangle = v_1 \vec{e}_1 + \ldots + v_n \vec{e}_n$.

WE CAN USE VECTORS TO DENOTE PHYSICAL QUANTITIES.

EX FORCE

- MULTIPLE FORCES ACTING ON AN OBJECT
- NET FORCE ON OBJECT (OR RESULTANT FORCE) IS SUM OF FORCES





$\vec{T}_1, \vec{T}_2$ TENSION FORCES

$\vec{W}$ = WEIGHT

$\vec{W} + \vec{T}_1 + \vec{T}_2 = \vec{0}$ (NEWTON — OBJECT IS AT REST)

DOT PRODUCT (§12.3)

$\vec{u} = \langle u_1, \dots, u_n \rangle$, $\vec{v} = \langle v_1, \dots, v_n \rangle$ THEN

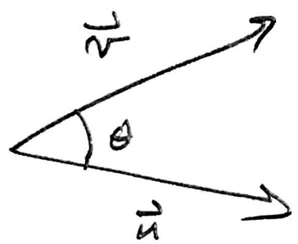
[i] $$\vec{u} \cdot \vec{v} = u_1 v_1 + \dots + u_n v_n = \sum_{j=1}^{n} u_j v_j .$$

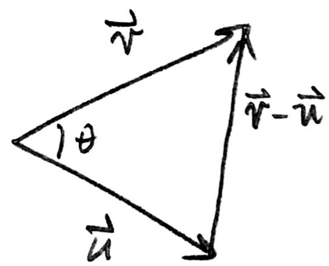
PROPERTIES: $\vec{u} = \langle u_1, \dots, u_n \rangle$, $\vec{v} = \langle v_1, \dots, v_n \rangle$, $c \in \mathbb{R}$

$$\left.\begin{array}{l} \bullet\ \vec{u} \cdot \vec{u} = |\vec{u}|^2 \\ \bullet\ \vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u} \\ \bullet\ \vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w} \\ \bullet\ (c\vec{u}) \cdot \vec{v} = c(\vec{u} \cdot \vec{v}) = \vec{u} \cdot (c\vec{v}) \end{array}\right\} \text{EASY FROM ADD, MULT ON } \mathbb{R}$$

IMPORTANT PROPERTY! $\vec{u}, \vec{v}$ VECTORS IN $\mathbb{R}^2$ OR $\mathbb{R}^3$, THEN

$$\vec{u} \cdot \vec{v} = \cos\theta\, |\vec{u}||\vec{v}| \qquad 0 \le \theta \le \pi \text{ angle between } \vec{u} \,\&\, \vec{v}$$





WHY? LAW OF COSINES

$\downarrow$

$$|\vec{v} - \vec{u}|^2 = |\vec{v}|^2 + |\vec{u}|^2 - 2|\vec{u}||\vec{v}|\cos\theta$$

PROPS: $$\begin{cases} (\vec{v} - \vec{u}) \cdot (\vec{v} - \vec{u}) = \vec{v} \cdot \vec{v} + \vec{u} \cdot \vec{u} - 2|\vec{u}||\vec{v}|\cos\theta \\ \underline{\vec{v} \cdot \vec{v}} - 2\vec{u} \cdot \vec{v} + \underline{\underline{\vec{u} \cdot \vec{u}}} = \underline{\vec{v} \cdot \vec{v}} + \underline{\underline{\vec{u} \cdot \vec{u}}} - 2|\vec{u}||\vec{v}|\cos\theta \end{cases}$$

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta \ \checkmark$$

NOTE: $\vec{u} \cdot \vec{v} = 0 \underset{\text{(IFF)}}{\iff} \theta = \pi/2 \iff \vec{u} \,\&\, \vec{v}$ ARE ORTHOGONAL (PERPENDICULAR) $\dots \perp$.

$$\iff \vec{u} \perp \vec{v}$$

[ii]