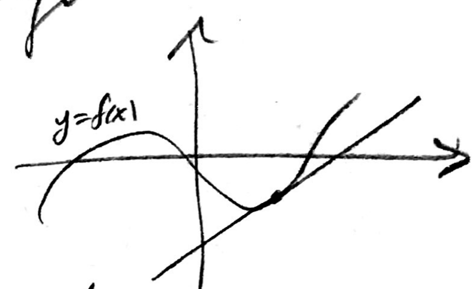
Math 241 – Lecture 1

Calculus 1 & 2: techniques for studying real valued functions, $y = f(x)$, of a single real variable.

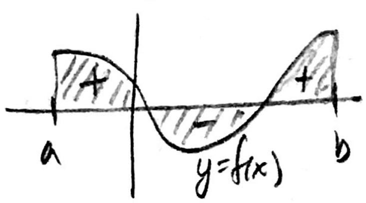
Three key tools/concepts:

① **Derivative**: • slope of tangent line to graph
• rate of change

$f'(x)$, $\frac{df}{dx}$, $\frac{dy}{dx}$, $\frac{d}{dx}f$



② **Integral**: • signed area under graph



$\int_a^b f(x)\,dx$

• average value

$\frac{1}{b-a}\int_a^b f(x)\,dx$

③ **Fundamental Theorem of calculus**:

relates ① & ②

$$f(b) - f(a) = \int_a^b f'(x)\,dx$$

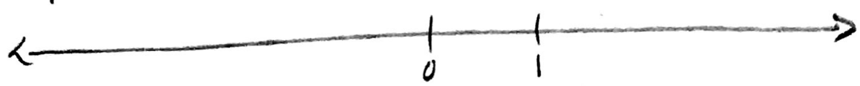
Too contstrained for the eal world –

Eg: ① Temperature depends on location and time: On Earth, need to specify longitude, latitude, time – 3 # input

② Your location at any given time is specified by long./lat. – 2 # output

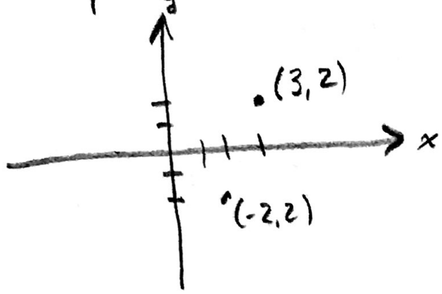
n-dimensional space (§12.1)

- 1-dim'l space = real line = real numbers = $\mathbb{R} = \mathbb{R}^1$



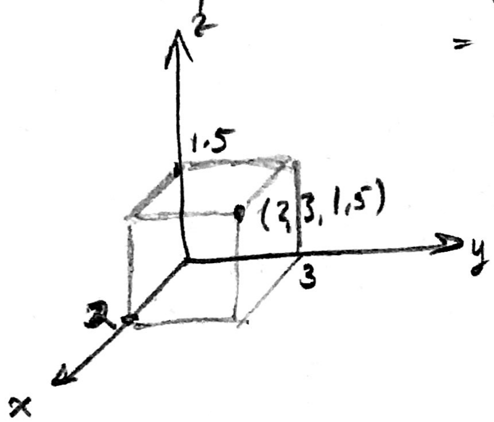
- 2-dim'l space = $\mathbb{R}^2$ = (cartesian) plane = $\underbrace{\{(x,y) \mid x,y \in \mathbb{R}\}}_{\text{notation!}}$

  = ordered pairs of real numbers.



- 3-dim'l space = $\mathbb{R}^3 = \{(x,y,z) \mid x,y,z \in \mathbb{R}\}$

  = ordered tuples of real numbers.



- n-dim'l space = $\{(x_1,\ldots,x_n) \mid x_1,\ldots,x_n \in \mathbb{R}\}$

  = ordered n-tuples of real numbers.

  = $\mathbb{R}^n$

We will use $\mathbb{R}^2$ and $\mathbb{R}^3$ to build intuition, but most (not all!) tools are applicable in $\mathbb{R}^n$.

Plan for the course

Develop calculus for functions w/ domain and/or range in $\mathbb{R}^n$:

1. Derivatives §14 (+13)
2. Integrals §15 (+13, 16)
3. Fundamental Theorems of calculus §16.

Need to better understand $\mathbb{R}^n$ (§12).

Question What sets calculus apart from algebra, trig, precalc?

Answer: Limits!

Recall: $\lim_{x \to a} f(x) = L$ means:

"as x approaches a, $f(x)$ approaches L".

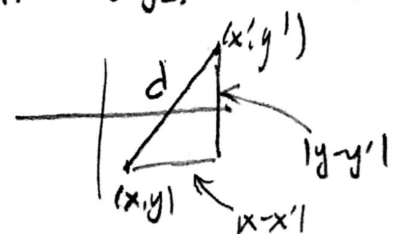
This requires a notion of proximity, i.e. distance.

Def'n Given $P = (x_1, x_2, \ldots, x_n)$, $Q = (y_1, \ldots, y_n) \in \mathbb{R}^n$,

$|PQ|$ = distance between P and Q

$$= \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots + (x_n - y_n)^2}$$

$n=2$ $\sqrt{(x-x')^2 + (y-y')^2}$

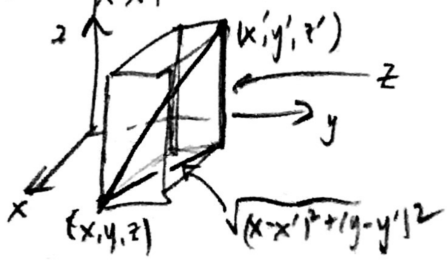


$d^2 = |x-x'|^2 + |y-y'|^2$

OR

$d = \sqrt{(x-x')^2 + (y-y')^2}$

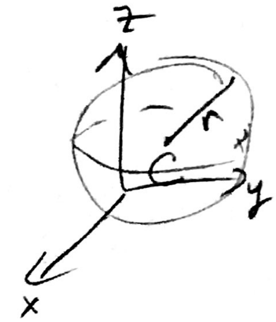
$n=3$ $\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$



Spheres

Defn: The sphere of radius $r > 0$ in $\mathbb{R}^3$ w/ center $C = (a,b,c)$ is the set of points in $\mathbb{R}^3$ w/ distance r to C.

$$\{ P \in \mathbb{R}^3 \mid |PC| = r \}$$



$P = (x,y,z)$, then:

$$|PC| = r \iff \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r$$
$$\iff \boxed{(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2}$$

Equation of a sphere center (a,b,c) radius r.
(set of solutions (x,y,z) to equation is sphere)

[[More equations for other surfaces later]]

In 1-variable, derivatives and integrals are defined via limits and arithmetic (and interactions)

In $\mathbb{R}^n$, "arithmetic" involves vectors (§12.2) - next time.

$\boxed{\text{syllabus}}$ - google LEININGER MATH ILLINOIS

- Read it! HW0 due Wednesday is on syllabus, WA
- Diary
  - notes
  - worksheets + solutions
  - daily info
- Piazza: discussion forum
- Disc. section - go! (T, Th)
- i>clicker - next time, dropped, ...
  - register!
- regrades / excused absences - see webpage, talk to TA.