Last time: critical points

Let $f : \mathbb{R}^2 \to \mathbb{R}$. A point $(a, b) \in \mathbb{R}^2$ is a critical point of f if one of the following holds:

1. $\nabla f(a, b) = \langle 0, 0 \rangle$; or
2. $\nabla f(a, b)$ is not defined.

Consider the function $f(x, y) = x \sin y$. Find all of its critical points (a, b). How many of them have $0 \leq b < 2\pi$?

(a) 1

(b) 2

(c) 3

(d) Infinitely many.

If you're finished, try to see if any of the critical points you found are local maxima or minima.

Announcements

- Midterm 1 graded and returned. Requests for regrade should be submitted in writing to your section TA, who will refer your question to the TA who graded that specific question.
- Homework deadline is at 8am. It will be strict starting next Monday!
- Please register your i-clicker. If you don't see your scores on Moodle, send me an email with your name, your UIN, and your i-clicker registration number.

Local maximum/minimum

Fix $f : \mathbb{R}^2 \to \mathbb{R}$, not necessarily differentiable; fix $(a, b) \in \mathbb{R}^2$.

- We say f has a local maximum at (a, b) if

$$f(a, b) \geq f(x, y) \text{ for all } (y, x) \text{ near } (a, b).$$

- We say f has a local minimum at (a, b) if

$$f(a, b) \leq f(x, y) \text{ for all } (y, x) \text{ near } (a, b).$$

Here "near (a, b)" means "for all (x, y) contained in a small disk of radius ϵ around the point (a, b)". (ϵ can be very small!)

Practice with the second derivative test

Recall the function $f(x, y) = x \sin y$. The point $(0, \pi)$ is a critical point. Find D.

(a) D = 0

(b) D = 1

(c) D = -1

(d) I don't know what to do.

Second derivative test

1. $D > 0, f_{xx}(a, b) > 0 \Rightarrow$ local minimum at (a, b).

2. $D > 0, f_{xx}(a, b) < 0 \Rightarrow$ local maximum at (a, b).

3. $D < 0 \Rightarrow$ saddle point at (a, b).