Problem Set 6

Q1 Let X be a normed space and $A \subset X$. Show that A is (strongly) bounded if and only if A is weakly bounded.
(By definition, A is weakly bounded if for each $f \in X^*$, there is a constant $M(f) > 0$ such that $f(a) \leq M(f)$ for all $a \in A$.)

Q2 Let X be a normed space. Show that $B(X)$ is w–closed and that $B(X^*)$ is w^*–closed.

Q3 Let X be a normed space and $Y \subset X$ be a closed subspace. Show that Y is also weakly closed.

Q4 Verify Claims 1, 2 and 3 in the proof of Alaoglu's theorem.

Q5 Let X be a normed space. Show that if X is reflexive, then $B(X)$ is w–compact.

Q6 Let X be a normed space and $f_1, \ldots, f_n \in X^*$ such that

$$\bigcap_{k=1}^{n} \ker f_k = \{0\}.$$

Show that $\dim X \leq n$.

Q7 Show that every weakly convergent sequence in l_1 is also (strongly) convergent. Does this imply that the topologies are the same?