The University of Melbourne

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAM for Math 620-413, Complex Analysis

Name (CAPITALS):____

Student Number:____________________

This exam is due back at or before 12:00 PM on Tuesday, November 6 2007. Late submissions will not be accepted. You may use textbooks or course notes when completing this exam, but you may not collaborate with other students or use any on-line resources. You may write your answers on other paper as long as it is attached to the exam paper when you hand it in and as long as the location of your answer is clearly indicated on the exam paper itself.

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DECLARATION:

- *I declare that all work submitted for this exam is my own work and does not involve plagiarism.*

Signed:____

Date:__________________________

1. (14 points) Let $f(z) = \frac{az+b}{cz+d}$ be a fractional linear transformation where a, b, c, and d are all real numbers with $c \neq 0$ and $ad - bc = 1$.

a. Find the image of the line $\{\mathrm{Re}(z) = -d/c\} \cup \{\infty\}$ under f.

b. The image of the line $\{\mathrm{Im}(z) = 1\} \cup \{\infty\}$ under f is a circle; find its diameter.

2. (18 points) Determine the set of all biholomorphic mappings from the standard sector $S_\alpha = \{z \in \mathbb{C} : \ 0 < \arg z < \alpha\}$, where $0 < \alpha < 2\pi$, onto the interior of the unit disc $D^2 = \{z \in \mathbb{C} : \ |z| < 1\}$.

3. (12 points) Find, with proof, the number of zeros of the following polynomials inside the given sets:

a. $2z^5 - z^3 + 3z^2 - 8z + 1$, $\{|z| < 1\}$

b. $z^4 - 4z^3 + 6z^2 - 4z + 3$, $\{|z-1| < 1\}$

c. $z^4 - 80z + 81$, $\{|z| < 3\}$

4. (10 points) Let $U \subset \mathbb{C}$ be a domain and $\mathcal{F}$ be a family of holomorphic functions which is normal in U. Prove that the family $\mathcal{G} = \{f' : f \in \mathcal{F}\}$ is normal in U.

5. (14 points) Find the power series expansion (i.e., find a general formula for the coefficients) for the function $f(z) = \frac{z-1}{z^2-2z+2}$ around the point $z = 1$. What is the radius of convergence of this power series?

6. (12 points) Assume that $p\colon S \to R$ is a covering map between compact Riemann surfaces, and that the genus of S is seven. What are the possibilities for the genus of R?

7. (20 points) Let S be a compact Riemann surface. A sequence of functions (f_n) on S is said to *converge normally* to a limit function f if the induced maps on coordinate charts converge uniformly on compact subsets.

Let $p\colon S \to R$ be a covering map. Show that if $\varphi_n\colon S \to S$ is a sequence of covering transformations which converges normally to the covering transformation $\varphi\colon S \to S$, then there is an index n_0 such that $\varphi_n = \varphi$ whenever $n \geq n_0$.