INDECOMPOSABLE NON-ORIENTABLE PD_3-COMPLEXES

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Abstract. We show that the orientable double covering space of an indecomposable, non-orientable PD_3-complex has torsion free fundamental group.

Let X be an indecomposable PD_3-complex, with fundamental group π and orientation character w. In [6] we showed that if $w \neq 1$ and π is virtually free then X is homotopy equivalent to $S^2\tilde{\times}S^1$ or $RP^2 \times S^1$, and so $\pi \cong \mathbb{Z}$ or $\mathbb{Z} \oplus Z/2Z$. In particular, $\mathrm{Ker}(w)$ is torsion free. We shall show that this remains true if $w \neq 1$ and π is *not* virtually free. This result is surely well-known for 3-manifolds. We give a short proof for this case in §1, which uses the "projective plane theorem" of [3] and a result from [6]. (The fact that RP^2 does not bound provides a further restriction here which is not yet known in general.) Our main result is Theorem 4 in §2. By passing to Sylow subgroups of the torsion in π, we may reduce potential counter-examples to special cases, which are eliminated by Lemma 3. (This lemma occupies almost half of the paper.) The arguments are similar to those of [6].

In the first two sentences of the Introduction to [6] we cited Crisp [1] inaccurately, overlooking the qualification "orientable" in his result on indecomposable orientable PD_3-complexes. Although the discussion of non-orientable PD_3-complexes in §7 of [6] begins by referring to the virtually free case, it would have been better to have reiterated this restriction in the formulation of Theorem 7.4 and its corollary. The final section gives some emendations to [6]. I would like to thank B.Hanke for alerting me to the necessity of considering the present case.

It is convenient to use the following notation and terminology. Let X^+ be the orientable covering space, with fundamental group $\pi^+ = \mathrm{Ker}(w)$. If $H \leq \pi$ then $H^+ = H \cap \pi^+$, and H is *orientable* if $H = H^+$. Let $Z/2Z^-$ denote a subgroup of order two on which $w \neq 1$. A graph of groups $(\mathcal{G}, \Gamma)$ is *admissible* if it is reduced, all vertex groups are finite or one-ended groups and all edge groups are nontrivial finite groups.

2000 *Mathematics Subject Classification.* Primary 57N13.

Key words and phrases. PD_3-complex, non-orientable, virtually free.

1. 3-manifolds

The result is relatively easy (and no doubt well-known) in the case of irreducible 3-manifolds, as we may use the Sphere Theorem, as strengthened by Epstein [3].

Theorem 1. *Let M be an indecomposable, non-orientable 3-manifold with fundamental group π. If π has infinitely many ends then $\pi \cong \pi^+ \rtimes Z/2Z^-$ and π^+ is torsion free, but not free.*

Proof. Let $\mathcal{P}$ be a maximal set of pairwise non-parallel 2-sided projective planes in M, and let $\mathcal{P}^+$ be the corresponding set of non-parallel 2-spheres in M^+. These sets are nonempty, since M is indecomposable and π has infinitely many ends. In particular, $\pi \cong \pi^+ \rtimes Z/2Z^-$, since the inclusion of a member of $\mathcal{P}$ splits $w = w_1(M) : \pi \to Z/2Z$. The components of $M^+ \setminus \mathcal{P}^+$ each double cover a component of $M \setminus \mathcal{P}$. Each such component is indecomposable [3].

Suppose that $M \setminus \mathcal{P}$ has a component Y with virtually free fundamental group. Then the double DY is indecomposable (cf. Lemma 2.4 of [6]), non-orientable and $\pi_1(DY)$ is virtually free. Moreover, $\pi_1(DY) \cong \mathbb{Z} \oplus Z/2Z^-$, since the inclusion of a boundary component of Y splits w. (See Theorems 7.1 and 7.4 of [6].) But then $DY \cong RP^2 \times S^1$, and so $Y \cong RP^2 \times [0,1]$. This is contrary to the hypothesis that the members of $\mathcal{P}$ are non-parallel. Thus the components of $M \setminus \mathcal{P}$ are punctured aspherical 3-manifolds.

Let Γ be the graph with vertex set $\pi_0(M \setminus \mathcal{P})$ and edge set $\mathcal{P}$, with an edge joining contiguous components. Then $\pi^+ \cong G * F(s)$, where G is a free product of PD_3-groups (corresponding to the fundamental groups of the components of $M \setminus \mathcal{P}$), and $s = \beta_1(\Gamma)$. Hence π^+ is torsion free. $\square$

We remark also that each component Y of $M \setminus \mathcal{P}$ has an even number of boundary components, since $\chi(\partial Y)$ is even (for any odd-dimensional manifold Y), by Poincaré duality. Thus the vertices of the graph Γ have even valence.

2. PD_3-complexes

Suppose now that X is an indecomposable PD_3-complex, with fundamental group π and orientation character w. Then π is finitely presentable, and so $\pi \cong \pi\mathcal{G}$, where $(\mathcal{G}, \Gamma)$ is an admissible graph of groups.

We wish to adapt the results from §7 of [6] to the cases when π has infinitely many ends and $w \neq 1$.

Lemma 2. *Let X be an indecomposable, non-orientable PD_3-complex with $\pi = \pi_1(X) \cong \pi\mathcal{G}$, where $(\mathcal{G}, \Gamma)$ is an admissible graph of groups. If $\pi = \pi_1(X)$ has nontrivial torsion then $\pi \cong \pi^+ \rtimes Z/2Z^-$. If π has no odd torsion then all finite vertex groups are non-orientable 2-groups and all edge groups are $Z/2Z^-$.*

Proof. If π is virtually free the assertions follow from Theorems 7.1 and 7.4 of [6]. Thus we may assume that $(\mathcal{G}, \Gamma)$ has at least one infinite vertex group and at least one edge.

If e is an edge of Γ and $w(g) = 1$ for all $g \in G_e$ of prime order then $G_{o(e)}$ and $G_{t(e)}$ are finite [1]. Thus if $G_{o(e)}$ or $G_{t(e)}$ is infinite then $G_e = Z/2Z^-$, and the inclusion of G_e into π splits w, so $\pi \cong \pi^+ \rtimes Z/2Z^-$.

If $G_{o(e)}$ and $G_{t(e)}$ are finite 2-groups then $N_\pi(G_e)$ is infinite, since $(\mathcal{G}, \Gamma)$ is reduced. Hence $G_e = Z/2Z^-$ again. Since Γ is connected it follows easily that every finite vertex group is non-orientable and every edge group is $Z/2Z^-$. □

Lemma 3. *Let X be an indecomposable PD_3-complex with $\pi = \pi_1(X) \cong \kappa \rtimes V$, where κ is orientable and torsion free and V is finite, of order $2p$, for some prime p. Then X is orientable.*

Proof. Suppose that X is not orientable. Then we may assume that $\pi \cong \pi\mathcal{G}$, where $(\mathcal{G}, \Gamma)$ is an admissible graph of groups with $r \geq 1$ finite vertex groups and at least one edge. Let $s = \beta_1(\Gamma)$.

Let $f : \pi \to V$ be a projection with kernel κ. Since f maps each finite vertex group injectively and $(\mathcal{G}, \Gamma)$ is reduced, it follows easily that all finite vertex groups are isomorphic to V. The edge groups must be $Z/2Z^-$, and if p is odd and there is an edge e with $G_{o(e)}$ and $G_{t(e)}$ both finite then V cannot be cyclic, for otherwise the normalizer of G_e in π would contain a non-abelian free group. However, vertices with finite vertex group are not necessarily connected by edges through other such vertices, and so we cannot yet conclude that V must be dihedral.

Since X^+ is the connected sum of aspherical PD_3-complexes with a PD_3-complex with virtually free fundamental group, $\pi^+ = G * F(s) * P$, where G is a free product of PD_3-groups and P is a free product of r copies of Z/pZ. Since $P = F(t) \rtimes Z/pZ$ (where $t = (p-1)(r-1)$, by a simple virtual Euler characteristic argument), $\kappa \cong G * F(s) * F(t)$. Let $a \in \pi$ be such that $a^2 = 1$ and $w(a) = -1$, and let $\lambda \cong \kappa \rtimes Z/2Z^-$ be the subgroup generated by κ and a. Then λ is also the group of a PD_3-complex, since it has finite index in π. The involution of π^+ induced by conjugation by a maps each noncyclic indecomposable factor to a conjugate of an isomorphic factor [4]. However, its behaviour on the free factor $F(s)$ may be more complicated.

The Fox-Lyndon Jacobian matrix associated to a presentation of a group π is a presentation matrix for the augmentation $\mathbb{Z}[\pi]$-module I_π. Its rows and columns are indexed by the generators and relators, respectively, and the element in the (i,j) position is the free derivative of the jth relator with respect to the ith generator. Thus the rows of the Fox-Lyndon presentation matrices for I_π and I_λ correspond to the generators of G, $F(s)$, P (or $F(t)$) and a, while the columns correspond to the relators for G, the action of a on G, the action of a on $F(s)$, the relators for P (or none), the action of a on P (or on $F(t)$) and the relation $a^2 = 1$. Let $w : \mathbb{Z}[\pi] \to R = \mathbb{Z}[a]/(a^2-1)$ be the linear extension of the orientation character. We may factor out the action of π^+ by tensoring with R. The images of the Jacobian matrices are then compatibly partitioned matrices of the form

$$C_\pi = \begin{pmatrix} M_1 & M_2 & M_3 & 0 & 0 & 0 \\ 0 & 0 & N_3 & 0 & 0 & 0 \\ 0 & 0 & T_\pi & U_4 & U_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & a+1 \end{pmatrix},$$

for $I_\pi \otimes R$, and

$$C_\lambda = \begin{pmatrix} M_1 & M_2 & M_3 & 0 & 0 \\ 0 & 0 & N_3 & 0 & 0 \\ 0 & 0 & T_\lambda & V & 0 \\ 0 & 0 & 0 & 0 & a+1 \end{pmatrix},$$

for $I_\lambda \otimes_w R$. The bottom right 2×3 and 2×2 blocks are the Fox-Lyndon presentation matrices for $I_{\pi/\gamma} \otimes_w R$ and $I_{\lambda/\gamma} \otimes_w R$, respectively, where γ is the normal subgroup of π generated by $G \cup F(s)$. Since these modules may be obtained from $I_\pi \otimes_w R$ and $I_\lambda \otimes_w R$ by factoring out the generators corresponding to the generators of G and $F(s)$ (or $F(t)$), the blocks T_π and T_λ must be linearly dependent on the blocks directly to the right. Hence we may assume that $T_\pi = 0$ and $T_\lambda = 0$.

Let H be the R-module with presentation matrix $\left(\begin{smallmatrix} M_1 & M_2 & M_3 \\ 0 & 0 & N_3 \end{smallmatrix}\right)$, the top left corner common to C_π and C_λ. Then

$$I_\pi \otimes_w R = H \oplus (I_{\pi/\gamma} \otimes_w R)$$

and

$$I_\lambda \otimes_w R = H \oplus (I_{\lambda/\gamma} \otimes_w R).$$

The groups P and its normal subgroup $F(t)$ have presentations

$$P = \langle b_i,\ 1 \le i \le r \mid b_i^p = 1,\ \forall i \rangle$$

and

$$F(t) = \langle x_{i,j},\ 1 \le i \le r-1,\ 1 \le j \le p-1 \mid \ \rangle,$$

where $x_{i,j}$ has image $b_1^j b_{i+1}^{-j}$ in P, for $1 \leq i \leq r-1$ and $1 \leq j \leq p-1$. (If $p = 2$ we shall write x_i instead of $x_{i,1}$, for$1 \leq i \leq r-1$.)

Suppose first that p is odd. If V is abelian then it has an unique element of order 2. Since $\pi/\gamma \cong P \rtimes Z/2Z^-$ and is the fundamental group of a graph of groups with vertex groups V and edge groups $Z/2Z^-$, we see that $\pi/\gamma \cong P \times Z/2Z^-$, and so $\lambda/\gamma \cong F(t) \times Z/2Z^-$. Hence

$$I_{\pi/\gamma} \otimes_w R \cong (R/(p, a-1))^r \oplus \widetilde{\mathbb{Z}}$$

and

$$I_{\lambda/\gamma} \otimes_w R \cong \mathbb{Z}^t \oplus \widetilde{\mathbb{Z}},$$

where $\mathbb{Z} = R/(a-1)$ and $\widetilde{\mathbb{Z}} = R/(a+1)$. The quotient ring $R/pR = \mathbb{F}_p[a]/(a^2-1)$ is semisimple, and so p-torsion R-modules have unique factorizations as sums of simple modules. Since $I_\pi \otimes_w R$ and $I_\lambda \otimes_w R$ satisfy Turaev's criterion (and projective R-modules are $\mathbb{Z}$-torsion free), the p-torsion submodule of $I_\pi \otimes_w R$ has equally many summands of types $R/(p, a-1)$ and $R/(p, a+1)$, and similarly for $I_\lambda \otimes_w R$. Since $I_{\lambda/\gamma} \otimes_w R$ is p-torsion free, the number of summands of types $R/(p, a-1)$ and $R/(p, a+1)$ in H must also be equal. On the other hand, $I_{\pi/\gamma} \otimes_w R$ has $r > 0$ summands of type $R/(p, a-1)$ and none of type $R/(p, a+1)$. These conditions are inconsistent, and so π is not the group of a non-orientable PD_3-complex.

Otherwise V has an unique conjugacy class of elements of order 2, and $\pi/\gamma \cong P \rtimes Z/2Z^-$ and $\lambda/\gamma \cong F(t) \rtimes Z/2Z^-$ have presentations

$$\langle a, b_i,\ 1 \leq i \leq r \mid a^2 = 1,\ b_i^p = 1,\ ab_ia = b_i^{-1}\ \forall i \rangle,$$

and

$$\langle a, x_{i,j},\ 1 \leq i \leq r-1,\ 1 \leq j \leq p-1 \mid a^2 = 1,\ ax_{ij}a = x_{i,p-j}\ \forall i, j \rangle,$$

respectively. (In particular, $\lambda/\gamma \cong F(t/2) * Z/2Z^-$.) In this case

$$I_{\pi/\gamma} \otimes {}_wR \cong (R/(p, a+1))^r \oplus \widetilde{\mathbb{Z}}$$

and

$$I_{\lambda/\gamma} \otimes_w R \cong R^{t/2} \oplus \widetilde{\mathbb{Z}}.$$

Consideration of the p-torsion submodules again shows that $I_\pi \otimes_w R$ and $I_\lambda \otimes_w R$ cannot both satisfy Turaev's criterion, and hence that π is not the group of a non-orientable PD_3-complex.

Suppose now that $p = 2$. The inclusions of the edge groups split w, by Lemma 2. In this case $V \cong (Z/2Z)^2 = Z/2Z \oplus Z/2Z^-$ and has two orientation reversing elements. The quotient π/γ is the group of a finite graph of groups with all vertex groups V and edge groups

$Z/2Z^-$. The underlying graph is a tree, since P is a free product of cyclic groups. Hence π/γ has a presentation

$$\langle a, b_i,\ 1 \leq i \leq r \mid a^2 = 1,\ b_i^2 = (aw_i)^2 = (aw_ib_i)^2 = 1,\ \forall i\rangle,$$

where $w_i = 1$ and $w_i \in F(t)$ for $2 \leq i \leq r$. The free subgroup $F(t)$ has basis $\{x_i | 1 \leq i \leq r-1\}$, where x_i has image b_1b_{i+1} in P, and λ/γ has a presentation

$$\langle a, x_i,\ 1 \leq i \leq r-1 \mid a^2 = 1,\ ax_ia = x_ib_{i+1}w_{i+1}b_{i+1}w_{i+1}^{-1},\ \forall i\rangle.$$

In this case

$$I_{\pi/\gamma} \otimes_w R \cong (R/(2, a-1))^r \oplus \widetilde{\mathbb{Z}}$$

and

$$I_{\lambda/\gamma} \otimes_w R \cong \mathbb{Z}^{r-1} \oplus \widetilde{\mathbb{Z}}.$$

Since $R/(2, a+1) = R/(2, a-1)$, torsion considerations do not appear to help. If $r > 1$ we may instead compare the quotients by the $\mathbb{Z}$-torsion submodules, as in Lemma 7.3 of [6], since finitely generated torsion free R-modules are direct sums of copies of R, $\mathbb{Z}$ and $\widetilde{\mathbb{Z}}$, by Theorem 74.3 of [2]. We again conclude that π is not the group of a non-orientable PD_3-complex.

The case when $p = 2$ and $r = 1$ requires a little more work. Let N be the R-module presented by the transposed conjugate of $\left(\begin{smallmatrix} 2 \\ a-1 \end{smallmatrix}\right)$. If $\{e, f\}$ is the standard basis for R^2 then $N = R^2/R(2e + (a+1)f)$. The $\mathbb{Z}$-torsion submodule of N is generated by the image of $(a-1)e$, and has order 2, but is not a direct summand. The quotient of N by its $\mathbb{Z}$-torsion submodule is generated by the images of e and $f - e$, and is a direct sum $\mathbb{Z} \oplus \widetilde{\mathbb{Z}}$. In particular, it has no free summand. It now follows easily that $H \oplus \widetilde{\mathbb{Z}} \oplus R/(2, a-1)$ is not stably isomorphic to $H \oplus \widetilde{\mathbb{Z}} \oplus N$. Therefore I_π and I_λ cannot both satisfy Turaev's criterion, and so π is not the group of a non-orientable PD_3-complex. □

We may now give our main result.

Theorem 4. *Let X be an indecomposable, non-orientable PD_3-complex with fundamental group π. If π has infinitely many ends then $\pi \cong \pi^+ \rtimes Z/2Z^-$ and π^+ is torsion free, but not free.*

Proof. The first assertion follows from Lemma 2, since π has infinitely many ends. Since $\pi^+ \cong G_0 * G_1$, where G_0 is virtually free and G_1 is a free product of PD_3-groups, there is a map $f : \pi \to J$ to a finite group J, with orientable, torsion free kernel κ.

Let $\pi \cong \pi\mathcal{G}$, where $(\mathcal{G}, \Gamma)$ is an admissible graph of groups. Suppose that there is a finite vertex group. Then there is a non-orientable finite vertex group, G_v say. Let $S(p)$ be a p-Sylow subgroup of G_v. Then

$f^{-1}f(S(2))$ has finite index in π, and has a graph of groups structure in which all finite vertex groups are 2-groups, and one vertex group is $S(2)$. If $S(2)$ is non-orientable then there is a $g \in S(2)$ such that $g^2 = 1$ and $w(g) = -1$, by Lemma 2.

Suppose now that $S(p) \neq 1$, for some odd prime p. Then $S \leq G_v^+$, and so is cyclic, since G_v^+ has periodic cohomology. The Sylow subgroup is characteristic in G_v^+, unless $p = 3$ or 5 and $G_v^+ = T^* \times Z/dZ$ or $I^* \times Z/dZ$. Thus S is normalized by g, and so G_v has a non-orientable subgroup V of order $2p$, except possibly when $p = 3$ or 5. Consideration of the possible involutions of T^* and I^* (as in Chapter 11 of [5]) shows that the only possible exception is if $p = 3$ and $G_v = I^* \times Z/dZ$. If we can show that no such vertex group has a subgroup of order $2p$ with p odd then the case $p = 5$ will exclude the possibility that $G_v^+ \cong I^* \times Z/dZ$ also, since $|I^*| = 2^3.3.5$.

The subgroup $f^{-1}f(V)$ has finite index in π, and is again the group of a non-orientable PD_3-complex. We now pass to an indecomposable factor whose group has V as one of its finite vertex groups. This factor is a non-orientable PD_3-complex with fundamental group $\kappa \rtimes V$. But this contradicts Lemma 3.

If $S(2)^+ \neq 1$ (i.e., if G_v^+ is a nontrivial 2-group) it has periodic cohomology, and so has an unique central involution. Hence G_v has a finite index subgroup $V \cong Z/2Z \times Z/2Z^-$. As before, passage to $f^{-1}f(V)$ leads to a contradiction, by Lemma 3.

Therefore all vertex groups are one-ended, and so π^+ is torsion free but not free. □

We expect that if $\pi = \pi\mathcal{G}$, where $(\mathcal{G}, \Gamma)$ is an admissible graph of groups, then all vertices of Γ have even valence. In the manifold case this follows from Poincaré duality. If v is a vertex of Γ then G_v^+ is a PD_3-group. Let E_v be the set of edges abutting on v. Is $(G_v, \{G_e : e \in E_v\})$ a PD_3-group pair? Can Turaev's splitting theorem be adapted to this situation? It remains possible that every indecomposable, non-orientable PD_3-complex is homotopy equivalent to a 3-manifold.

We conclude by restating Corollary 7.5 of [6], which is an immediate consequence of [1] and Theorem 4.

Corollary 5. *Let X be a PD_3-complex and $g \in \pi = \pi_1(X)$ a nontrivial element of finite order. If $C_\pi(g)$ is infinite then g is an orientation-reversing involution and $C_\pi(g) = \langle g \rangle \times Z$.* □

Are there any examples other than $RP^2 \times S^1$ of indecomposable PD_3-complexes whose groups have an involution with infinite centralizer?

3. EMENDATIONS OF EARLIER WORK

We take this opportunity to make some emendations to [6], in particular, to the results (in §7) relating to the non-orientable case.

In Theorem 3.1, DC_* should be defined by $DC_q = \overline{Hom_{\mathbb{Z}[\pi]}(C_{3-q}, \mathbb{Z}[\pi])}$, for all q.

In the proof of Theorem 5.2, the third sentence of the first paragraph should be "If 4 divides $|G_e|$ then G_e has a central involution, which is also central in $V = G_{o(e)}$ and $W = G_{t(e)}$, since these groups have cohomological period 4. (See the remarks preceding Lemma 2.1.)"

In the third paragraph, d should be k, say, as it is NOT the d of the statement, and the final sentence should be "Since the odd-order subgroup of G_e is central in W its normalizer in V must be abelian. Hence either $k = 3$ and $V = B \times Z/dZ$ with $B = T_1^*$ or I^* and $(d, |B|) = 1$ or $k = 1$, by Lemma 5.1."

Minor improvements have been made to the next two statements.

Lemma (7.3)**.** *Let X be an indecomposable PD_3-complex with $\pi = \pi_1(X) \cong F(r) \rtimes G$, where G is finite. If π has an orientation reversing element g of finite order then G has order $2m$, for some odd m. Hence π has an orientation reversing involution.* □

In the first paragraph of the proof of Lemma 7.3 we may and should also reduce to the case when G is abelian, of order a multiple of 4. (See the penultimate paragraph of Theorem 4 above.) The rest of the proof then goes through as written.

Theorem (7.4)**.** *Let X be an indecomposable, non-orientable PD_3-complex with $\pi = \pi_1(X)$ virtually free. If π has an orientation reversing involution then $X \simeq S^1 \times RP^2$.* □

As the statement of Theorem 7.4 now assumes that π is virtually free, the first sentence of the proof should be deleted. In the final sentence of the second paragraph of the proof, the vertex groups must all be D_{2p}, by the normalizer condition and Crisp's Theorem (and so $\varepsilon = -1$, later in the proof). In the presentation, a_i should be a (no subscript). The argument otherwise needs no change.

Corollary 7.5 only follows from Theorem 7.4 if π is virtually free, but holds in general by Corollary 5 above. (In fact, Crisp's original result already implies that $C_\pi(g) \cong \langle g \rangle \times \mathbb{Z}$ or $\langle g \rangle \times D_\infty$, since every element of the maximal finite normal subgroup of a group with two ends has infinite centralizer.)

Finally, some typos:
Statement of Theorem 5.2: "$\mathbb{Z}2\mathbb{Z}$" should be "$\mathbb{Z}/2\mathbb{Z}$".

Second paragraph of §6: b_1 should be b_n in the last relator of the presentation.

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