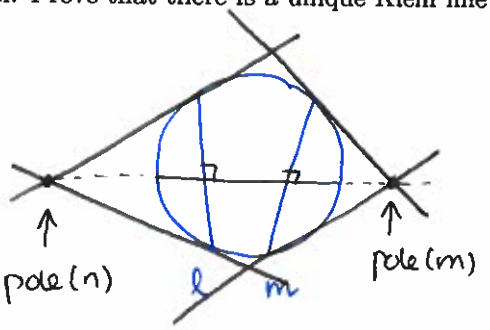
MATH 402 Practice questions

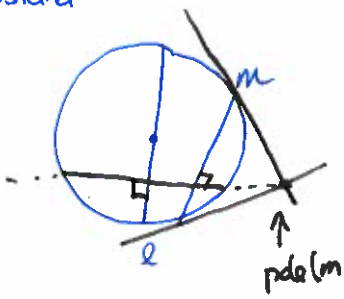
Friday 2 November, 2018

Exercise 1. Hyperbolic geometry: Let ℓ and m be two Klein lines which are parallel but not limiting parallel. Prove that there is a unique Klein line n which is perpendicular to both ℓ and m.



CASE 1 – no diameters

Make sure you understand all terms used.



CASE 2 – 1 diameter

Exercise 2. Checking something is an isometry:

(a) What is the definition of an isometry $f : \mathbb{R}^2 \to \mathbb{R}^2$?

$AB = f(A)f(B) \quad \forall \quad A, B \in \mathbb{R}^2$.

(b) Review the proof that any isometry can be written as a composition of at most three reflections.

See lecture notes

(c) For each of the following functions, is it an isometry? Prove or disprove.

$f(x, y) = (y, x)$ – yes (reflection)

$g(x, y) = (x + y, y + a)$ – ~~yes~~ no

$h(x, y) = (x + a, y + b)$ – yes (translation)

$j(x, y) = (-y, x)$ – yes. (rotation)

Exercise 3. Using the classification of isometries to identify isometries:
Consider the following functions of the Euclidean plane. For each, indicate in the table whether it is possible that the function is a reflection, (non-identity) rotation, (non-identity) translation, glide reflection (with non-zero displacement vector), or the identity. For this problem, assume that ℓ and m are two lines and O is a point on ℓ.

	Reflection	Rotation	Translation	Glide reflection	Identity
$f = r_\ell \circ r_m \circ r_\ell$	✓				
An isometry f which satisfies $f(O) = O$	✓	✓			✓
An even isometry		✓	✓		✓
The composition of a glide reflection and a translation	✓			✓	
The function $f(x, y) = (2x, y)$					
An isometry f which has ℓ as its *only* invariant line				✓	
An isometry f which satisfies $f^3 = \mathrm{id}$		✓			✓
An isometry f which can be represented by a 2×2 matrix	✓	✓			✓
An isometry f which has no fixed points			✓	✓	
An isometry f which is the square of a glide reflection			✓		