MATH 402 Homework 5

Due Friday October 13, 2017

Exercise 1. a. *[3 pts]* Let $T = r_{\ell_2} \circ r_{\ell_1}$ be a translation, with displacement vector v. Prove that the inverse of T is also a translation, given by $r_{\ell_1} \circ r_{\ell_2}$ and having displacement vector $-v$.

b. *[3 pts]* Let T_1 and T_2 be two translations, with displacement vectors v_1 and v_2 respectively. Prove that $T_1 \circ T_2$ is again a translation. What is its displacement vector?

c. *[3 pts]* Show that composition of translations *commutes*: that is, that $T_1 \circ T_2$ is equal to $T_2 \circ T_1$. Is this true for reflections? Prove or provide a counter-example.

d. *[3 pts]* Does the set of translations form a group?

Exercise 2. *[10 pts]* Let T be a translation which is not the identity. Prove that ℓ is an invariant line for T if and only if ℓ is parallel to the displacement vector v of T.

Exercise 3. a. *[8 pts]* Suppose we are given a coordinate system with origin O. Let Rot_ϕ denote rotation about O by angle ϕ. Let $C = (x, y)$ be a point not equal to O, and let T denote the translation with displacement vector $v = (x, y)$. Prove that $T \circ Rot_\phi \circ T^{-1}$ is rotation about C by angle ϕ.

b. *[8 pts]* Given a coordinate system with origin O, let ℓ be a line which does not pass through O. Using translations, rotations, and reflection across the x-axis, give an expression for reflection r_ℓ across ℓ.

Exercise 4. a. *[2 pts]* Let Rot_ϕ be rotation about a point O by angle ϕ. Use reflections to prove that the inverse of Rot_ϕ is rotation about O by angle $-\phi$.

b. *[3 pts]* Let Rot_ψ be rotation about the same point O by angle ψ. Use reflections to prove that $Rot_\phi \circ Rot_\psi$ is again a rotation about O.

c. *[4 pts]* Let A and B be two different points. Let R_1 be rotation about A by $180°$, and let R_2 be rotation about B by $180°$. Prove that $R_2 \circ R_1$ is a translation. What is the displacement vector?

d. *[3 pts]* Let $\mathcal{R}$ denote the set of all rotations. Let $\mathcal{R}_O$ denote the set of all rotations with centre of rotation O. Is $\mathcal{R}$ a group? What about $\mathcal{R}_O$?

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.