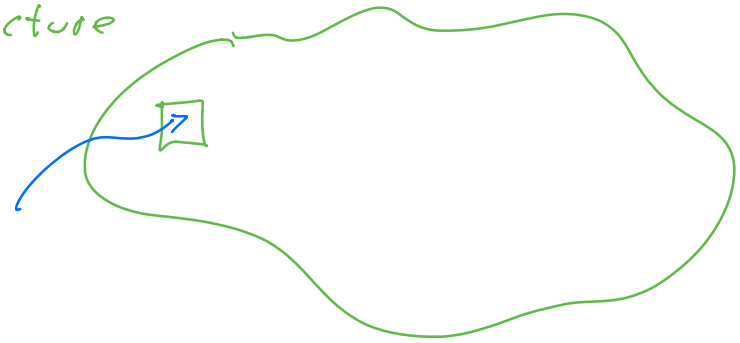
II - A simple dynamical System

2.1 Motivation

Picture



Carrots $N_C \equiv$ Number of squares filled by Carrots

Rabits $N_R \equiv$ Number of " " " Rabits

The quantity of interest:

$$x = \frac{N_R}{N_R + N_C}$$, fraction of squares that are rabits.

$0 \leq x \leq 1$

How x evolves from day t to day $t+1$?

(discrete time dynamical system)

$$x_{t+1} = F(x_t)$$

Conditions on F:

- $F(0) = 0$
- $F(1) = 0$
- $F(x) = ax$, $a > 1$, for $x \gtrsim 0$
- Symmetry $x \to 1-x$, $F(x) = F(1-x)$

Simplest case is $F(x) = ax(1-x)$

$$x_{t+1} = a\, x_t\, (1 - x_t)$$

Given x_0, x at $t=0$, we

Example $a = 2$

$x_0 = 0.811979\ldots$

$x_1 = 2 \cdot x_0 (1 - x_0)$

$\vdots$

$x_{10} =$

Results:

$x_{10} = 0.5$