Harmonic Analysis and Related Areas Special Session

Australian Mathematical Society Annual Meeting
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Titles and abstracts of the talks

Speakers

- Dooley
- Duong
- Hall
- Laca
- Varghese
- Wright

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Wavelets on manifolds: a group theoretic approach

Tony Dooley (University of New South Wales)

The theory of wavelets on $\mathbb{R}^n$ seems to depend critically on the existence of dilations and appropriate discrete subgroups. It is less clear how to proceed when $\mathbb{R}^n$ is replaced by a Lie group or a homogeneous space. I shall give a brief survey of the relevant parts of the Euclidean thoery before describing some recent work which provides an approach to this problem.

Boundedness of singular integral operators on L^p spaces

Xuan Thinh Duong (Macquarie University)

Let T be a bounded operator from $L^2(\Omega)$ to $L^2(\Omega)$ where Ω is a subset of the Euclidean space $\mathbb{R}^n$ or $\mathbb{R}^n$ itself. The question is to find a sufficient condition on the kernel of T so that T is bounded on $L^p(\Omega)$, $1 < p < \infty$.

In the first part of the talk, we will explain the Calderón-Zygmund decomposition on $\mathbb{R}^n$ and the Hörmander condition which is sufficient for T to be of weak type $(1, 1)$. By interpolation, T is bounded on L^p for $1 < p \leq 2$.

In the second part, we will present a sufficient condition for T to be of weak type $(1, 1)$. Our condition is weaker than the usual Hörmander condition. We also extend the result to the case when $\mathbb{R}^n$ is replaced by a subset Ω with no assumptions on the smoothness of its boundary.

We will give applications of our conditions to various problems such as functional calculi of operators, Riesz transforms on manifolds and maximal regularity for abstract Cauchy problems.

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Statistical aspects of wavelet methods

Peter Hall (Centre for Mathematics and its Applications, A.N.U.)

Thresholding is used in statistical applications of wavelet methods to achieve a balance between systematic and stocastic error—or equivalently to achieve a balance between bias and variance. However, commonly used forms of thresholding do not achieve this balance in a particularly optimal way; they tend to produce estimators that are oversmoothed, i.e., which have too much systematic error. Recent new developments, based on thresholding in blocks, offer better performance in this respect. The talk will review statistical aspects of wavelet methods, with particular reference to choice of thresholds and resolution levels.

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A semigroup crossed product approach to phase transitions from number theory

Marcelo Laca (University of Newcastle)

I will begin with an introduction to C^*-dynamical systems as models for quantum statistical dynamics, including a discussion of equilibrium states and phase transitions. I will then show how to construct a class C^*-dynamical systems based on semigroup crossed products. The equilibrium states of these systems are more symmetric than one would expect and they can be characterized by a Markov rescaling condition on probability measures. This will allow us to construct and analyze easy examples of systems exhibiting phase transitions. More interestingly, these methods also provide a simplified approach to the phase transitions with spontaneous symmetry breaking in number theory of Bost and Connes.

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Twisted group C^*-algebras and positive scalar curvature

Mathai Varghese (University of Adelaide)

Recall that one of the simplest invariants of the Riemann curvature tensor R is the scalar curvature function, $k : M \to \mathbb{R}$, which is defined in terms of an orthonormal basis $(e_1, \ldots, e_n)$ at T_xM to be $k(x) = -\sum_{i,j=1}^{n} \langle R(e_i, e_j)\, e_i, e_j \rangle_x$ where $\langle , \rangle_x$ denotes the Riemannian metric on T_xM. When M has dimension 2, then the scalar curvature coincides with the Gauss curvature. The scalar curvature measures how quickly the volume of the ball of radius r is growing when compared to a ball of radius r in Euclidean space, as $r \to 0$. In particular, it follows that a Riemannian manifold has positive scalar curvature if and only if the volume of small radii balls on M is dominated by the volume of the corresponding small radii balls in Euclidean space. The scalar curvature also appears in the Einstein-Hilbert action functional in General Relativity, where it is important to know when there exist metrics of positive scalar curvature. Therefore one can pose the following:

Fundamental Question: *Under what topological conditions does a compact oriented manifold M (of dimension ≥ 5), admit a Riemannian metric of positive scalar curvature?*

This question has been studied by many mathematicians, notably Lichnerowicz, Connes, Gromov, Lawson, Rosenberg and Stolz (amongst others), who have made great progress. One of the remaining cases which has not been completely settled is when M is a general non simply-connected compact spin manifold, and there is a conjecture due to Gromov-Lawson-Rosenberg in this case. I will discuss a 'twisted' approach to studying this question using twisted group C^*-algebras, where I establish a (known) special case of this conjecture but using the new method. More precisely, I will outline an approach to compute the K-theory of twisted group C^*-algebras and under certain assumptions, together with a twisted L^2 index theorem for covering spaces one can compute the range of the trace on K-theory in terms of classical characteristic classes on the the classifying space $B\Gamma$. This is most interesting in the particular case of two dimensions, when Γ is a discrete cocompact subgroup of $PSL(2,\mathbb{R})$, where it is treated in my earlier work (jointly with others) and in the case of $\mathbb{Z}^2$, it is treated in detail by Rieffel and Pimsner-Voiculescu. As a consequence of this result, it can be shown that in this case,

(*i*) one can establish that there are only a finite number of projections in the twisted group C^*-algebra whenever the multiplier is a root of unity (i.e. rational);

(*ii*) one can classify the twisted group C^*-algebras up to isomorphism;

(*iii*) one can show that projectively periodic elliptic operators (which are self adjoint and semi-bounded below) on any Γ-covering space have only a finite number a spectral gaps in any half-line $(-\infty,\lambda]$, whenever the multiplier is a root of unity. This is related to the generalised Ten Martini problem.

Uniform bounds for multidimensional oscillatory integrals and sublevel sets

Jim Wright (University of New South Wales)

Oscillatory integrals arise throughout analysis, especially harmonic analysis. In many problems, the main difficulty lies in obtaining appropriate bounds when one knows that some derivative of the phase of the oscillatory integral is large. In one dimension there is a satisfactory theory which give uniform bounds. We seek to obtain such uniform bounds in higher dimensions. Along the way, we'll run into some interesting combinatorial and plane geometry problems.

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